

# The Frequency-Response Design Method

Recall the frequency-response formula:

$$\sin(\omega t) \longrightarrow \boxed{G(s)} \longrightarrow M \sin(\omega t + \phi)$$

where  $M = M(\omega) = |G(j\omega)|$  and  $\phi = \phi(\omega) = \angle G(j\omega)$

Derivation:

1.  $u(t) = e^{st} \mapsto y(t) = G(s)e^{st}$
2. Euler's formula:  $\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$
3. By linearity,

$$\begin{aligned} \sin(\omega t) &\mapsto \frac{G(j\omega)e^{j\omega t} - G(-j\omega)e^{-j\omega t}}{2j} & G(j\omega) &= M(\omega)e^{j\phi(\omega)} \\ &= \frac{M(\omega)e^{j(\omega t + \phi(\omega))} - M(\omega)e^{-j(\omega t + \phi(\omega))}}{2j} \\ &= M(\omega) \sin(\omega t + \phi(\omega)) \end{aligned}$$