

Rule E: Asymptotes

How does the locus look as $s \rightarrow \infty$?

$$\begin{aligned} 180^\circ = \angle L(s) &= \angle \frac{s^m + b_1 s^{m-1} + \dots}{s^n + a_1 s^{n-1} + \dots} \\ &= \angle \frac{s^{m-n} + b_1 s^{m-n-1} + \dots}{1 + a_1 s^{-1} + \dots} \\ &\simeq \angle s^{m-n} \text{ if } |s| \rightarrow \infty \quad (\text{recall } m \leq n) \end{aligned}$$

Claim: If $\angle s^{m-n} = 180^\circ$, then

$$\angle s = \frac{180^\circ + \ell \cdot 360^\circ}{n - m}, \quad \ell = 0, 1, \dots, n - m - 1$$

Proof:

$$\begin{aligned} s &= |s|e^{j\angle s} & s^{m-n} &= |s|^{m-n}e^{j(m-n)\angle s} \\ (m-n)\angle s = 180^\circ & \implies & (m-n)\angle s &= 180^\circ + \ell \cdot 360^\circ \end{aligned}$$