Complete Root Locus

$$L(s) = \frac{s+1}{s(s+2)\big((s+1)^2+1\big)}$$

Rule A: 4 branches

Rule B: branches start at $p_1, \ldots, p_4$

Rule C: branches end at $z_1, \pm\infty$

Rule D: real locus $= [z_1, p_1] \cup (-\infty, p_2]$

Rule E: asymptotes form angles at $60°, 180°, -60°$

Rule F: $j\omega$-crossings at $\pm j\omega_0$, where

$$\omega_0 = \sqrt{1+\sqrt{5}} \approx 1.8$$
$$\text{when } K = 4\sqrt{5} \approx 8.9$$

(transition from stability to instability)

$60°$

$j\omega_0 = j\sqrt{1+\sqrt{5}}$
when $K = 4\sqrt{5}$

p_3

Im

Re

p_2 z_1 p_1

p_4

$-60°$