Rule F: $j\omega$-crossings

In our example, the characteristic polynomial is

$$s^4 + 4s^3 + 6s^2 + (4 + K)s + K$$

The critical value: $K = 4\sqrt{5}$ (from Routh test).

To find the $j\omega$-crossing, plug in and solve:

$$\begin{aligned} & (j\omega)^4 + 4(j\omega)^3 + 6(j\omega)^2 + (4 + 4\sqrt{5})j\omega + 4\sqrt{5} = 0 \\ & \omega^4 - 4j\omega^3 - 6\omega^2 + (4 + 4\sqrt{5})j\omega + 4\sqrt{5} = 0 \\ \text{real part:} \quad & \omega^4 - 6\omega^2 + 4\sqrt{5} = 0 \\ \text{imag. part:} \quad & -4\omega^3 + 4(1 + \sqrt{5})\omega = 0 \qquad \omega^2 = 1 + \sqrt{5} \end{aligned}$$

$j\omega$-crossing at $j\omega_0 = \sqrt{1 + \sqrt{5}} \approx 1.8$, when $K = 4\sqrt{5} \approx 8.9$