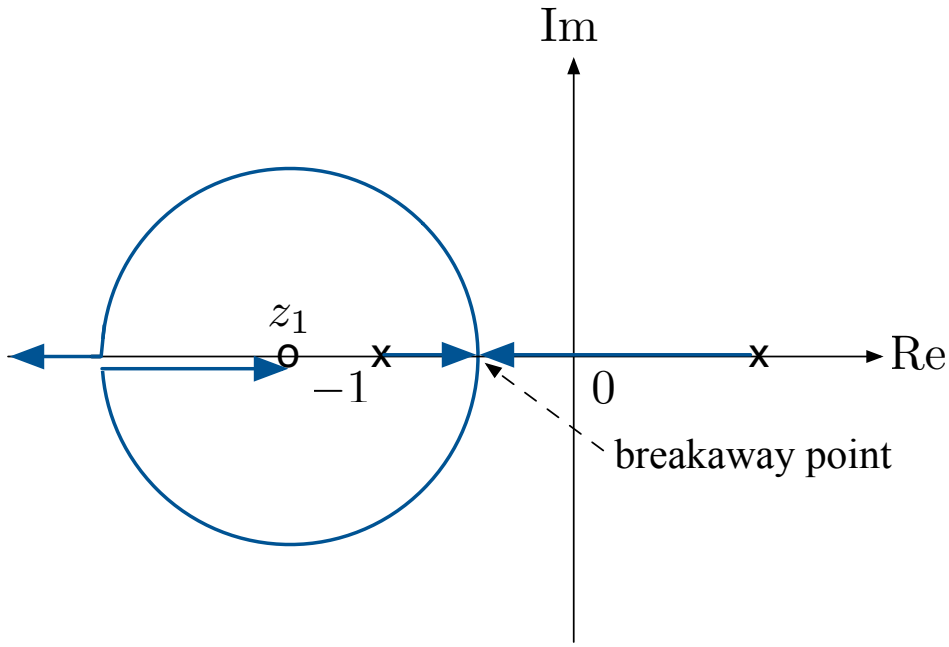
$$L(s) = \frac{s - z_1}{s^2 - 1}$$



Is the point $s = 0$ on the root locus?

Let's see if there is any value $K > 0$, for which this is possible:

$$1 + KL(0) = 0$$

$$1 + Kz_1 = 0 \qquad K = -\frac{1}{z_1} > 0 \text{ does the job}$$