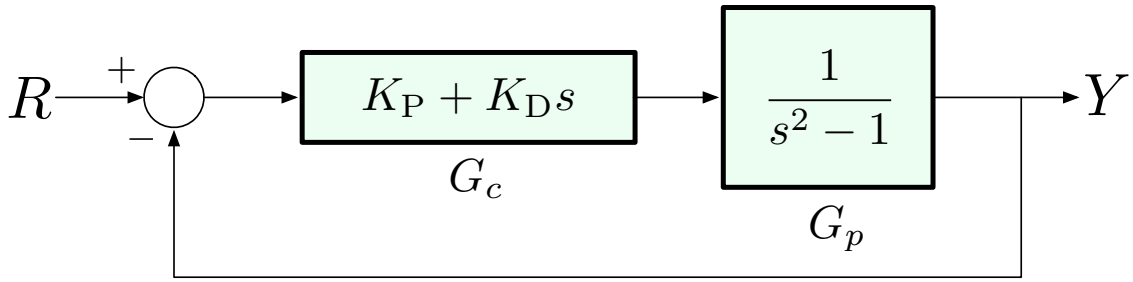
Example

PD control of an unstable 2nd-order plant



We will examine the impact of varying $K = K_\mathrm{D}$, assuming the ratio $K_\mathrm{P}/K_\mathrm{D}$ *fixed*.

Let us write the characteristic equation in *Evans form*:

$$1 + \underbrace{K_\mathrm{D}}_{K}\left(s + \frac{K_\mathrm{P}}{K_\mathrm{D}}\right)\left(\frac{1}{s^2-1}\right) = 1 + K\,\underbrace{\frac{s + K_\mathrm{P}/K_\mathrm{D}}{s^2-1}}_{L(s)} = 0$$

$$L(s) = \frac{s - z_1}{s^2 - 1} \qquad \text{zero at } s = z_1 = -K_\mathrm{P}/K_\mathrm{D} < 0$$