Equivalent Characterization of RL: Phase Condition

Recall our original definition: The *root locus* for $1 + KL(s)$ is the set of all closed-loop poles, i.e., the roots of

$$1 + KL(s) = 0,$$

as K varies from 0 to ∞.

A point $s \in \mathbb{C}$ is on the RL if and only if

$$L(s) = \underbrace{-\frac{1}{K}}_{\text{negative and real}} \quad \text{for some } K > 0$$

This gives us an equivalent characterization:

The phase condition: The root locus of $1 + KL(s)$ is the set of all $s \in \mathbb{C}$, such that $\angle L(s) = 180°$, i.e., $L(s)$ is real and negative.