

# Dirichlet Prior Smoothing

- Dirichlet distribution is a conjugate prior for multinomial sampling distribution

“pseudo” word counts  $\alpha_i = \mu p(w_i | \text{REF})$

$$\text{Dir}(\theta | \alpha_1, \dots, \alpha_N) = \frac{\Gamma(\alpha_1 + \dots + \alpha_N)}{\Gamma(\alpha_1) \dots \Gamma(\alpha_N)} \prod_{i=1}^N \theta_i^{\alpha_i - 1}$$

$$\times p(d | \theta) = \frac{|d|!}{c(w_1)! \dots c(w_N)!} \prod_{i=1}^N \theta_i^{c(w_i, d)}$$



$$p(\theta | d) = \text{Dir}(\theta | \alpha_1 + c(w_1), \dots, \alpha_N + c(w_N))$$

$$= \frac{\Gamma(\alpha_1 + \dots + \alpha_N + |d|)}{\Gamma(\alpha_1 + w_1) \dots \Gamma(\alpha_N + w_N)} \prod_{i=1}^N \theta_i^{c(w_i) + \alpha_i - 1}$$