

Divergence Minimization

$$\theta^* = \operatorname{argmin}_{\theta} \left[\frac{1}{n} \sum_{i=1}^n D(\theta || \theta_{D_i}) \right] - \lambda D(\theta || \theta_C) \quad \text{Subject to } \boxed{\sum_{w \in V} p(w|\theta) = 1}$$

$$\frac{\partial g(\theta)}{\partial \mu} = 0$$

$$\begin{aligned} \theta^* &= \operatorname{argmin}_{\theta} \left[\frac{1}{n} \sum_{i=1}^n H(\theta; \theta_{D_i}) - H(\theta) \right] - \lambda H(\theta; \theta_C) + \lambda H(\theta) & \theta_w &= p(w|\theta) \\ &= \operatorname{argmin}_{\theta} \frac{1}{n} \sum_{i=1}^n [H(\theta; \theta_{D_i}) - \lambda H(\theta; \theta_C)] - (1 - \lambda) H(\theta) \end{aligned}$$

$$g(\theta) = -\frac{1}{n} \sum_{i=1}^n [\sum_{w \in V} p(w|\theta) [\log p(w|\theta_{D_i}) - \lambda \log p(w|\theta_C)]] + (1 - \lambda) \sum_{w \in V} p(w|\theta) \log p(w|\theta) + \boxed{\mu [\sum_{w \in V} p(w|\theta) - 1]}$$

$$\frac{\partial g(\theta)}{\partial \theta_w} = -\frac{1}{n} \sum_{i=1}^n [\log p(w|\theta_{D_i}) - \lambda \log p(w|\theta_C)] + (1 - \lambda) [\log p(w|\theta) + 1] + \mu = 0$$

Solution:

$$\theta_w = p(w|\theta) \propto \exp \left[\frac{1}{1-\lambda} \frac{1}{n} \sum_{i=1}^n \log p(w|\theta_{D_i}) - \frac{\lambda}{1-\lambda} \log p(w|\theta_C) \right]$$