

# Why is exact inference intractable?

$$P(\mathbf{Z}, \Theta, \Phi | \mathbf{W}, \alpha, \beta) = \frac{P(\mathbf{W}, \mathbf{Z}, \Theta, \Phi | \alpha, \beta)}{P(\mathbf{W} | \alpha, \beta)}$$

where

$$\begin{aligned} P(\mathbf{W} | \alpha, \beta) &= \int_{\Phi} \int_{\Theta} \sum_{\mathbf{Z}} P(\mathbf{W}, \mathbf{Z}, \Theta, \Phi | \alpha, \beta) d\Theta d\Phi \\ &= \int_{\Phi} p(\Phi | \beta) \int_{\Theta} p(\Theta | \alpha) \sum_{\mathbf{Z}} p(\mathbf{Z} | \Theta) p(\mathbf{W} | \mathbf{Z}, \Phi) d\Theta d\Phi \end{aligned}$$

Denominator integral is intractable due to **coupling** between  $\theta$  and  $\phi$  in the summation over  $\mathbf{Z}$