Baum-Welch Algorithm

Basic "counters":

$$\gamma_t(i) = p(q(t) = s_i \mid O, \lambda)$$

Being at state s_i at time t

$$\xi_t(i,j) = p(q(t) = s_i, q(t+1) = s_j \mid O, \lambda)$$

Being at state s_i at time t and at state s_j at time t+1

Computation of counters:

$$\gamma_t(i) = \frac{\alpha_t(i)\beta_t(i)}{\sum_{j=1}^{N} \alpha_t(j)\beta_t(j)}$$

$$\xi_t(i,j) = \frac{\alpha_t(i) a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)}{\sum_{j=1}^{N} \alpha_t(j)\beta_t(j)}$$

$$= \gamma_t(i) \frac{a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)}{\beta_t(i)}$$

Complexity: $O(N^2)$