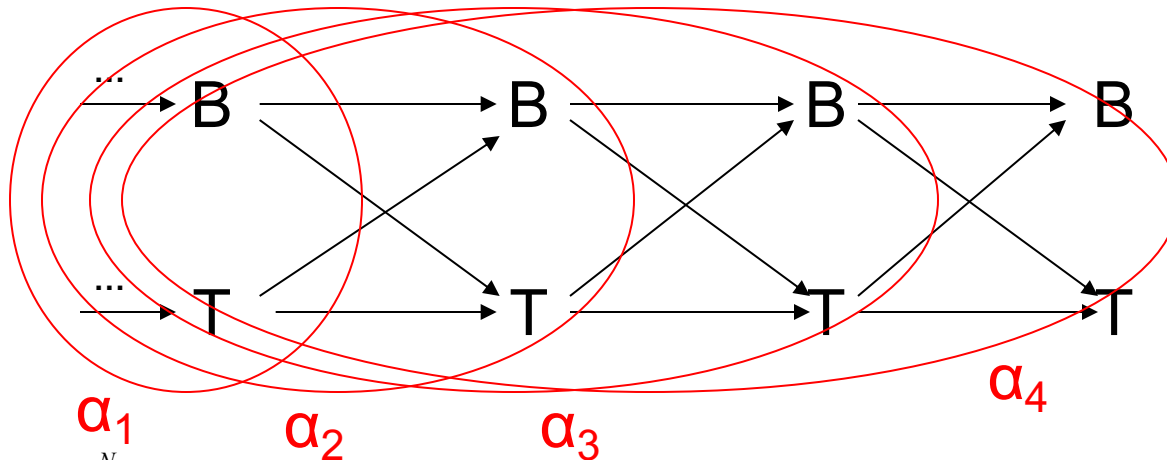


The Forward Algorithm



$$p(o_1 \dots o_T | \lambda) = \sum_{i=1}^N \sum_{s_1 s_2 \dots s_{T-1}} p(o_1 \dots o_T, s_1 s_2 \dots s_{T-1}, S_T = s_i)$$

$$\alpha_t(i) = \sum_{s_1 s_2 \dots s_{t-1}} p(o_1 \dots o_t, s_1 s_2 \dots s_{t-1}, S_t = s_i)$$

Generating $o_1 \dots o_t$
with ending state s_i

$$= \sum_{s_1 s_2 \dots s_{t-1}} p(o_1 \dots o_{t-1}, s_1 s_2 \dots s_{t-1}) p(S_t = s_i | S_{t-1}) p(o_t | S = s_i)$$

$$= \sum_{j=1}^N [\sum_{s_1 s_2 \dots s_{t-2}} p(o_1 \dots o_{t-1}, s_1 s_2 \dots s_{t-1} = s_j)] a_{ji} b_i(o_t)$$

$$= b_i(o_t) \sum_{j=1}^N \alpha_{t-1}(j) a_{ji}$$

The data likelihood is

$$p(o_1 \dots o_T | \lambda) = \sum_{i=1}^N \alpha_T(i)$$

Complexity: $O(TN^2)$