Viterbi Algorithm

Observation: $\max_{S_1 S_2 \ldots S_T} p(o_1 \ldots o_T, S_1 \ldots S_T) = \max_{s_i} [\max_{S_1 S_2 \ldots S_{T-1}} p(o_1 \ldots o_T, S_1 \ldots S_{T-1}, S_T = s_i)]$

Prob. of best path ending at state i

Algorithm:
(Dynamic programming)

$$VP_t(i) = \max_{S_1 \ldots S_{t-1}} p(o_1 \ldots o_t, S_1 \ldots S_{t-1}, S_t = s_i)$$

$$q_t^*(i) = [\arg\max_{S_1 \ldots S_{t-1}} p(o_1 \ldots o_t, S_1 \ldots S_{t-1}, S_t = s_i)] \rightarrow (i)$$

Best path ending at state i

1. $VP_1(i) = \pi_i b_i(o_1), \; q_1^*(i) = (i), \; for \; i = 1, \ldots, N$

2. $For \; 1 \le t < T, \; VP_{t+1}(i) = \max_{1 \le j \le N} VP_t(j) a_{ji} b_i(o_{t+1}),$

$$q_{t+1}^*(i) = q_t^*(k) \rightarrow (i), \; k = \arg\max_{1 \le j \le N} VP_t(j) a_{ji} b_i(o_{t+1}), \; for \; i = 1, \ldots, N$$

The best path is $q_T^*(i)$ Complexity: $O(TN^2)$