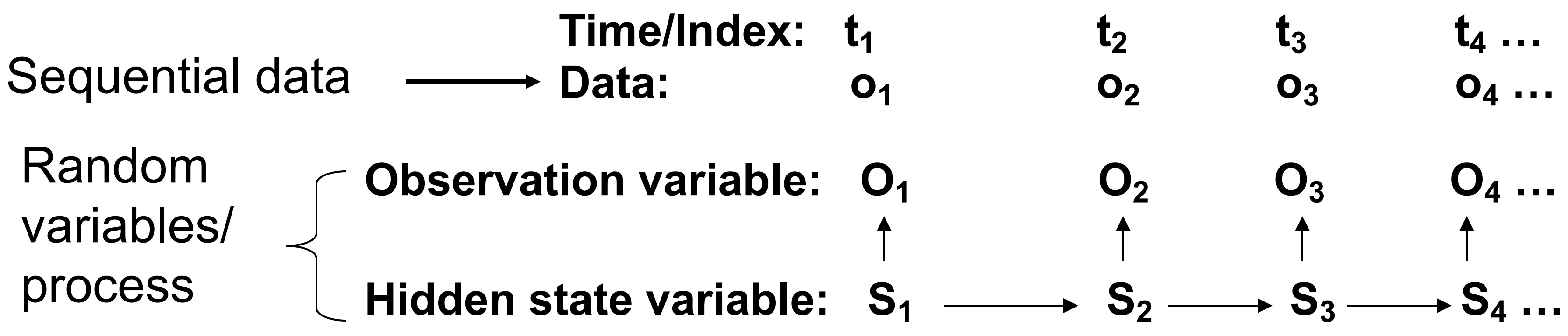
HMM as a Probabilistic Model



Sequential data → **Time/Index:** t_1 t_2 t_3 t_4 …

Data: o_1 o_2 o_3 o_4 …

Random variables/ process

Observation variable: O_1 O_2 O_3 O_4 …

Hidden state variable: S_1 S_2 S_3 S_4 …

State transition prob: $p(S_1,S_2,\ldots,S_T)=p(S_1)p(S_2\mid S_1)\ldots p(S_T\mid S_{T-1})$

Probability of observations with known state transitions:

$$p(O_1,O_2,\ldots,O_T\mid S_1,S_2,\ldots,S_T)=p(O_1\mid S_1)p(O_2\mid S_2)\ldots p(O_T\mid S_T)$$

Joint probability (complete likelihood):

$$p(O_1,O_2,\ldots,O_T,S_1,S_2,\ldots,S_T)=\overbrace{p(S_1)}^{\text{Init state distr.}}p(O_1\mid S_1)p(S_2\mid S_1)\overbrace{p(O_2\mid S_2)}^{\text{Output prob.}}\ldots\underbrace{p(S_T\mid S_{T-1})}_{\text{State trans. prob.}}p(O_TS_T)$$

Probability of observations (incomplete likelihood):

$$p(O_1,O_2,\ldots,O_T)=\sum_{S_1,\ldots S_T}p(O_1,O_2,\ldots,O_T,S_1,\ldots S_T)$$