$$\mathbf{w}_i^{\mathsf{T}} \tilde{\mathbf{w}}_j + b_i + \tilde{b}_j = \log X_{ij}$$

Again: two sets of vectors: "target" vectors ${\bf w}$ and "context" vectors ${\tilde {\bf w}}$.

 $X_{ij} = n_{w_i,w_j}$ is the number of times w_j appears in the context of w_i .

Problem: all co-occurrences are weighted equally

$$J = \sum_{i,j=1}^{V} f(X_{ij}) (\mathbf{w}_i^{\mathsf{T}} \tilde{\mathbf{w}}_j + b_i + \tilde{b}_j - \log X_{ij})^2$$

Final objective is just **weighted least-squares**. $f(X_{ij})$ serves as a "dampener", lessening the weight of the rare co-occurrences.

$$f(x) = \begin{cases} \left(\frac{x}{x_{max}}\right)^{\alpha} & \text{if } x < x_{max} \\ 1 & \text{otherwise} \end{cases}$$

where α 's default is (you guessed it) 3/4, and x_{max} 's default is 100.