

AVL Runtime Proof

An upper-bound on the height of an AVL tree is **$O(\lg(n))$** :

$N(h)$:= Minimum # of nodes in an AVL tree of height h

$$N(h) = 1 + N(h-1) + N(h-2)$$

$$> 1 + 2^{h-1/2} + 2^{h-2/2}$$

$$> 2 \times 2^{h-2/2} = 2^{h-2/2+1} = 2^{h/2}$$

Theorem #1:

Every AVL tree of height h has at least $2^{h/2}$ nodes.