Proving $f(n) \geq 2n$ for $n \geq 8$

Definition of function $f : \mathbb{Z}^+ \to \mathbb{Z}$:

- $f(1) = f(2) = 1$ and
- $f(n) = f(n-1) + f(n-2)$ for $n \geq 3$

We wish to prove $f(n) \geq 2n$ for $n \geq 8$.

Let $P(k)$ be the statement that $f(k) \geq 2k$.

Base cases: we already have shown $P(N)$ is true for $N = 8, 9, 10$.
Let $N \geq 10$ be arbitrary (note that I pick 10 because that is the largest base case I examined).
Our Inductive Hypothesis is that $P(k)$ is true for all $k, 8 \leq k \leq N$.