

Why would simple induction have failed?

Note that $g(N + 1)$ is defined in terms of $g(N - 1)$, so we need a statement that is true about $g(N - 1)$ and not just about $g(N)$.

In other words, $P(N)$ is not enough to establish $P(N + 1)$.

We also needed $P(N - 1)$.