

Proving closed form solution for $g(n)$

Since $N \geq 10$, $N + 1 \geq 11$.

Therefore, by the definition of the function g , we have:

$$g(N + 1) = g(N + 1 - 2) = g(N - 1)$$

Note that $9 \leq N - 1 < N$ so that by the Inductive Hypothesis, $P(N - 1)$ is true.

Hence $g(N - 1) = 1$ if $N - 1$ is odd and $g(N - 1) = 3$ if $N - 1$ is even.

But note that

- ▶ $N - 1$ and $N + 1$ have the same parity (both are odd or both are even)

Therefore, we have established that:

- ▶ $g(N + 1) = 1$ if $N + 1$ is odd and
- ▶ $g(N + 1) = 3$ if $N + 1$ is even

In other words, we have shown that $P(N + 1)$ is true.

Since $N \geq 10$ was arbitrary, $P(N)$ is true for all $n \in \mathbb{Z}^+$.