Proving $f(n) \geq 2n$ for $n \geq 8$

The Inductive Hypothesis is

- $P(8) \wedge P(9) \wedge \ldots \wedge P(N)$ is true

where $P(k)$ asserts that $f(k) \geq 2k$ and $N \in \mathbb{Z}^{\geq 10}$ is arbitrary.

Since $N \geq 10$, by the definition of the function f:

- $f(N+1) = f(N) + f(N-1)$

Note that $N-1$ and $N-2$ are both at least 8. Therefore, by the I.H.:

- $f(N) \geq 2N$ and
- $f(N-1) \geq 2(N-1)$

Hence $f(N+1) \geq 2N + 2(N-1) \geq 4N - 2 \geq 2(N+1)$

Note: the second inequality uses that $N \geq 10$.

Hence $f(N+1) \geq 2(N+1)$, which is the same as $P(N+1)$.

Since $N \geq 10$ was arbitrary, $P(N)$ is true for all $N \geq 8$.