Functions of two variables

Let $f : \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Z}^+$ be defined by

- $f(n, m) = n + m$ if $n = 1$ or $m = 1$,
- $f(n, m) = f(n - 1, m) + f(n, m - 1)$, otherwise

Why is $f(1, 3) = 4$?

- Because we use the first bullet to compute $f(1, 3)$, and we get $f(1, 3) = 1 + 3 = 4$

Why is $f(2, 2) = 6$?

- Because we use the second bullet to compute $f(2, 2)$, and we get $f(2, 2) = f(1, 2) + f(2, 1)$.
- Also, $f(1, 2) = 1 + 2 = 3$ and $f(2, 1) = 2 + 1 = 3$.
- Therefore $f(2, 2) = 3 + 3 = 6$.