

Summarizing what we did

Recall that we had a recursively defined function

$f : \mathbb{Z}^+ \times \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$, defined by

- ▶ $f(n, m) = n + m$ if $n = 1$ or $m = 1$,
- ▶ $f(n, m) = f(n - 1, m) + f(n, m - 1)$, otherwise

We wanted to prove that $f(m, n) \geq m + n$ for all positive integers m, n .

It is easy to verify this inequality for the case where $m = 1$ or $n = 1$. To prove it true for all m, n , we used induction.

But induction must be done for some single parameter.

We used $K = m + n$ as our single parameter.

Our inductive hypothesis was

$$P(K) : f(n, m) \geq n + m \text{ for all positive integers } n, m \text{ with } n + m \leq K$$