Another induction proof, continued

So far we have shown that when n, m are both at least 2 and $n + m = K + 1$, then

$$f(n, m) = f(n-1, m) + f(n, m-1) \geq 2(n + m - 1)$$

However, a little more arithmetic finishes this!

$$f(n, m) \geq 2(n + m - 1) = n + m + (n + m - 2) \geq n + m$$

since $n + m - 2 \geq 0$.

Since K was arbitrary, the statement holds for all $K \geq 2$, and hence for all pairs of positive integers n, m that sum to K.

This is what we wanted to prove. Q.E.D.