Another induction proof, continued

Let n, m be given so that $n + m = K + 1$.

If $n = 1$ or $m = 1$, then by definition $f(n, m) = n + m$, and the statement holds.
So assume $n \geq 2$ and $m \geq 2$, so that

$$f(n, m) = f(n-1, m) + f(n, m-1)$$

Note that $n + m = K + 1$ and so $n + m - 1 = K$.

Hence we can apply the Inductive Hypothesis to $f(n-1, m)$ and $f(n, m-1)$.

Therefore,

$$f(n-1, m) \geq n + m - 1$$

and

$$f(n, m-1) \geq n + m - 1$$

Hence

$$f(n, m) = f(n-1, m) + f(n, m-1) \geq 2(n + m - 1)$$