

Finishing the Induction Proof

Recall $f : \mathbb{Z}^+ \times \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$, and the inductive hypothesis is:

$$P(K) : f(n, m) \geq n + m \text{ for all positive integers } n, m \text{ with } n + m \leq K$$

To finish the induction proof, we need to show $P(K) \rightarrow P(K + 1)$, which is equivalent to showing

$$P(K) \rightarrow \forall n, m \text{ such that } n + m \leq K + 1, f(n, m) \geq n + m$$

However, since the I.H. assumes $f(n, m) \geq n + m$ whenever $n + m \leq K$, we only need to show that

- ▶ $P(K) \rightarrow f(n, m) \geq n + m$ whenever $n + m = K + 1$.