

Using induction to prove theorems about recursive functions of two variables

Let $f : \mathbb{Z}^+ \times \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ be defined by

- ▶ $f(n, m) = n + m$ if $n = 1$ or $m = 1$,
- ▶ $f(n, m) = f(n - 1, m) + f(n, m - 1)$, otherwise

We would like to prove that $f(n, m) \geq n + m$ for all $n \geq 1$ and $m \geq 1$.

Base cases: $n = 1$ or $m = 1$ follows immediately. So we prove the rest by induction.

What is our inductive hypothesis?