Proof by contradiction that $\sqrt{7}$ is irrational, continued

Recall we assumed that $\sqrt{7}$ is rational and so then $\exists a, b$ relatively prime integers such that $(\frac{a}{b})^2 = 7$.

Therefore $a^2 = 7b^2$ (Arithmetic)

Hence 7 divides a^2 (Equality of a^2 and $7b^2$, and 7 divides $7b^2$)

Because 7 is prime, this implies that 7 divides a

But then 7^2 divides a^2 (obvious)

and so 7^2 divides $7b^2$ (because $a^2 = 7b^2$)

And so 7 divides b^2 (obvious)

Because 7 is prime, this implies that 7 divides b

Hence 7 is a common divisor of both a and b, which contradicts our earlier assumption.