

# Proving theorems about graphs

Theorem: Every finite simple graph has an even number of vertices of odd degree.

Recall  $SUM$  (sum of degrees) is even.

Let  $ODD$  denote the set of vertices of odd degree and  $EVEN$  denote the set of vertices of even degree.

$$SUM = \sum_{v \in ODD} \deg(v) + \sum_{v \in EVEN} \deg(v)$$

Since  $SUM$  is even and  $\sum_{v \in EVEN} \deg(v)$  is even,  $\sum_{v \in ODD} \deg(v)$  must also be even.

But then  $|ODD|$  is even!

Q.E.D.