Proving $A = \emptyset$

Recall that we assume A is a finite set of integers and satisfies $x \in A \to x + 1 \in A$. We first show that A cannot be non-empty, and then we show that $A = \emptyset$ satisfies the constraint.

- If A is finite but non-empty, then it has n elements, and so $A = \{x_1, x_2, \dots, x_n\}$. Let y be the maximum element of A. Since $y \in A$, it follows that $y + 1 \in A$.
  But $y + 1$ is bigger than every element of A, which is a contradiction. Hence, A cannot be non-empty.
- On the other hand, does $A = \emptyset$ satisfy the required property:
  - If $x \in A$ then $x + 1 \in A$

  Yes, because an "IF-THEN" statement is true whenever the first half is false. And $x \in \emptyset$ is always false.