Proof by contradiction that $\sqrt{7}$ is irrational, continued

Here's a longer justification for why 7 must divide a if 7 divides a^2.

Every integer greater than 1 has a unique prime factorization.

So let the prime factorization of a be:

$$a = \prod_{i=1}^{k} p_i^{l_i}$$

where p_i is a prime and l_i is a positive integer.

Hence the unique prime factorization of a^2 must be

$$a^2 = \prod_{i=1}^{k} p_i^{2l_i}$$

If 7 divides a^2 then $7 = p_i$ for some $i, 1 \leq i \leq k$. (the definition of saying that 7 divides a^2)

Hence 7 is one of the prime factors for a, and so 7 divides a.