Proofs by induction that go wrong

Let G = (V, E) be a simple graph with N vertices with minimum degree at least 1.

By the inductive hypothesis, G is connected.

Let G' be the graph formed by adding one vertex, x, to G.

If G' has minimum degree at least 1, then x must be adjacent to some vertex y.

Since y is connected (via a path) by every other vertex in G, x is also connected (via a path) to every other vertex in G'.

Hence G' is connected.

Since G was arbitrary, this proves that all simple finite graphs with minimum degree 1 are connected.