Finishing the proof

We showed that

$$P(1) \wedge P(2) \wedge \ldots P(n) \rightarrow P(n+1)$$

whenever $n \geq 2$.

We also had established that $P(1)$ and $P(2)$ were true.

Hence, $P(n)$ is true for all $n \in \mathbb{Z}^+$.

In other words, we have shown that $F(n) = n \mod 2$ for all $n \in \mathbb{Z}^+$.

Q.E.D.