Why weak induction can fail

Let $F : \mathbb{Z}^+ \to \mathbb{Z}$ be defined by

- $F(1) = 1$ and $F(2) = 0$
- $F(n) = F(n-2)$ if $n > 2$

We assume $n \geq 2$ is arbitrary and $P(n)$ is true.

Then $n + 1 \geq 3$ and so (by definition) $F(n+1) = F(n-1)$.

What's our next step?

Recall the Inductive Hypothesis is $P(n)$: $F(n) = n \mod 2$.

The Inductive Hypothesis tells us nothing about $F(n-1)$.

What do we do now???