Another simple proof by induction

Let A_n for $n \in \mathbb{Z}^+$ be a set defined by:

- $A_1 = \emptyset$
- $A_n = A_{n-1} \cup \{n-1\}$

We wish to prove that $A_n = \{1, 2, \ldots, n-1\}$ for all $n \in \mathbb{Z}^{\geq 2}$.

Let's prove this by induction!

Base case is $n = 2$.
By definition, $A_2 = A_1 \cup \{1\} = \emptyset \cup \{1\} = \{1\}$.
So the base case is good!

The Inductive Hypothesis is the statement
$P(n) =$ "$A_n = \{1, 2, \ldots, n-1\}$"

Let $n \geq 2$ be arbitrary.
Can we show $P(n) \rightarrow P(n+1)$?