Very easy induction proof

Let $N \geq 0$ be arbitrary.

Assume $P(N)$ is true. Hence, $f(N) = 0$.

Since $N \geq 0, N + 1 \geq 1$.
Hence, by definition, $f(N + 1) = f(N) = 0$.
In other words, we have shown:

$$P(N) \rightarrow P(N + 1)$$

Since N was arbitrary, $P(N)$ is true for all N.

In other words, we have shown that $f(n) = 0$ for all $n = 0, 1, 2, \ldots$.