Bounding this recurrence relation

The running time $t_2(n)$ for this algorithm satisfies

- $t_2(1) \leq C_0$
- $t_2(2) \leq C_1$
- $t_2(n) \leq t_2(n-1) + C_2$

for some positive constants C_0, C_1, and C_2.
Let $C' = \max\{C_0, C_1, C_2\}$. It is easy to see that $C' > 0$.

We can prove that $t_2(n) \leq C'n$ for all $n \geq 1$, by induction on n (i.e., linear time).

(Note: even simple induction suffices for this proof.)

In other words, this dynamic programming algorithm is linear time.

In contrast, the recursive algorithm for computing Fibonacci numbres was exponential time!