Proofs by contradiction

We are trying to prove that $\forall n \in \mathbb{Z}^+, P(n)$, where $P(n) \equiv [1 + 2 + \ldots + n = n(n+1)/2]$.

1. We showed $P(1)$ true and we let N be the smallest positive integer n such that $\neg P(n)$. Hence $N \geq 2$ and so $N - 1 \geq 1$.
2. Therefore, $P(N-1)$ is true, and so
$$1 + 2 + \ldots + (N-1) = (N-1)N/2$$
3. We add N to both sides of the equation above, and obtain
$$1 + 2 + \ldots + N = (N-1)N/2 + N$$
4. Note that
$$(N-1)N/2 + N = N(N+1)/2$$
so that $1 + 2 + \ldots + N = N(N+1)/2$
5. Thus, we have derived $P(N)$, contradicting our hypothesis!
6. Therefore, it must be that $\forall n \in \mathbb{Z}^+, 1 + 2 + \ldots + n = n(n+1)/2$.