

Proofs by contradiction

Theorem: $\forall n \in \mathbb{Z}^+, 1 + 2 + \dots + n = n(n + 1)/2$

Proof by contradiction.

Let $P(n)$ denote the assertion $1 + 2 + \dots + n = n(n + 1)/2$.

If the theorem isn't true, then $P(n)$ is not true for some $n \in \mathbb{Z}^+$.

Note that $P(1)$ is true.

Therefore the smallest n such that $P(n)$ is False must be at least 2.

Let's call the smallest such value N , so that $P(N)$ is False.

Since $N \geq 2$, it follows that $N - 1 \geq 1$ and so $P(N - 1)$ is True!