Problems

Problem 1: Let $F : \mathbb{Z}^+ \to \mathbb{Z}$ be defined recursively by

- $F(1) = 3$
- $F(n) = 2F(n-1) + 1$ if $n \geq 2$

Prove that $F(n) > 2^{n-1}$ for all $n \in \mathbb{Z}^+$.

Problem 2: Let A_n, $n \in \mathbb{Z}^+ \cup \{0\}$, be defined by

- $A_0 = \{0\}$, and
- $A_n = A_{n-1} \cup \{n^2\}$ if $n \geq 2$.

Prove that $A_n = \{i^2 | 0 \leq i \leq n, i \in \mathbb{Z}\}$ for all integers $n \geq 0$.