Same theorem, now proof by induction

Recall definition of S_n. We let $P(n)$ be the Boolean statement $S_n = \{x \in \mathbb{Z}^+ | x \leq n\}$"

Theorem: $P(n)$ is true for all $n \in \mathbb{Z}^+$
Proof: by induction on n.

- The **base case** is $n = 1$.
  By definition, $S_1 = S_0 \cup \{1\} = \{1\}$, and so $P(1)$ is true.
- Let $N \in \mathbb{Z}^+$ be arbitrary.
- **Inductive hypothesis:** $P(N)$ is true.
- Note $N + 1 \geq 2$, and so by definition $S_{N+1} = S_N \cup \{N + 1\}$.
- By the I.H., $S_N = \{1, 2, \ldots, N\}$.
- Hence $S_{N+1} = \{1, 2, \ldots, N\} \cup \{N + 1\} = \{1, 2, \ldots, N + 1\}$.
- Since N was arbitrary, $P(N)$ is true for all $N \geq 1$.