Proof by contradiction

Let $P(n)$ be the Boolean statement "$S_n = \{1, 2, \ldots, n\}$."

1. We verified that $P(1)$ is true, by noting that $S_1 = \{1\}$.
2. Now suppose it is not the case that $\forall n \in \mathbb{Z}^+, P(n)$. Let N be the smallest positive integer for which $\neg P(N)$. Note that $N > 1$ since $P(1)$ is true.
3. Hence, $N - 1 \geq 1$. Therefore, $P(N-1)$ must be true, and so
$$S_{N-1} = \{1, 2, \ldots, N-1\}$$
4. Since $N > 1$, by definition
$$S_N = S_{N-1} \cup \{N\}$$
5. Combining these two statements we get:
$$S_N = \{1, 2, \ldots, N-1\} \cup \{N\} = \{1, 2, \ldots, N\}$$
And so $P(N)$ is true.

But then this contradicts our hypothesis. Hence the theorem must be true.