Recursively defined sets

Just as functions are often defined recursively, so can sets be. Let's consider some recursively defined sets.

•
$$S_0 = \emptyset$$

• $S_n = S_{n-1} \cup \{n\}$ for $n \ge 1$.

Questions:

- 1. What is S_1 ? (Answer: $S_1 = S_0 \cup \{1\} = \{1\}$)
- 2. What is S_2 ?
- 3. What is a closed form formula for S_n ?
- 4. Can you prove your formula correct for all n?