Connecting proofs by contradiction and induction

We used "proof by contradiction" to show

$$\forall n \geq n_0, P(n)$$

1. We assumed the statement

   $$\forall n \geq n_0, P(n)$$

   is false, and so inferred there must be some smallest number $N \geq n_0$ for which $\neg P(N)$.

2. We showed $P(n_0)$ is true.

3. Hence $N > n_0$, and so $N - 1 \geq n_0$.

4. Since N is the smallest number greater than or equal to n_0 for which $P(N)$ is false, it must be that $P(N - 1)$ is true.

5. We then derived $P(N)$ is true, which contradicted our hypothesis.