Proving $F(n) \geq n$ for all $n \geq 2$ by contradiction

Recall the definition of the function $F : \mathbb{Z}^+ \to \mathbb{Z}^+$:

- $F(1) = 0$
- $F(n) = 2 + F(n-1)$ if $n \geq 2$

We want to prove that $\forall n \geq 2,\ F(n) \geq n$
We assumed N was the smallest positive integer such that $F(N) < N$ and showed

$$F(N-1) \geq N-1$$

Since $N \geq 3 > 2$, by definition

$$F(N) = 2 + F(N-1)$$

Combining these two statements, we get

$$F(N) \geq 2 + (N-1) = N + 1 > N$$

But this means $P(N)$ is true, contradicting our hypothesis.