Proving $F(n) \geq n$ for all $n \geq 2$ by contradiction

How small can N be?

Since $P(2)$ is true, it must be that $N \geq 3$.

Hence $N - 1 \geq 2$.

Since N is the smallest integer $n \geq 2$ for which $P(n)$ is false, $P(N - 1)$ must be true.

Hence

$$F(N - 1) \geq N - 1$$