

## Proof that every tree with $n$ vertices has $n - 1$ edges

Since  $T$  is a tree,  $T$  has at least two leaves.

Let  $v$  be a leaf in  $T$ , and let  $w$  be its single neighbor.

Let  $T'$  be the graph created by deleting  $v$ .

Note that  $T'$  is a tree with  $K$  vertices, because:

- ▶  $T'$  has one less vertex than  $T$ .
- ▶  $T'$  is connected and acyclic

By the inductive hypothesis,  $T'$  has  $K - 1$  edges.

Recall that  $T'$  has one less edge than  $T$ .

Hence  $T$  has  $K$  edges. (q.e.d.)

**Important:** We started with a tree on  $K + 1$  vertices and removed a leaf to get a tree on  $K$  vertices. We did not go the reverse direction!