Proving every tree with at least two vertices has at least two leaves

Proof by contradiction.
Let $T = (V, E)$ be a tree with at least two vertices, and let P be a longest path in tree T.
We write $P = v_1, v_2, \ldots, v_k$, with $k \geq 2$.

Suppose v_1 is not a leaf in T.
Then the neighbor set of v_1, denoted $\Gamma(v_1)$, has at least two vertices, and so $\exists x \in V, x \neq v_2$ such that $(v_1, x) \in E$.

- If x is in the path P, then $x = v_i$ for some $i > 2$, and $v_1, v_2, \ldots, v_i, v_1$ is a cycle in T, which is a contradiction (because T is a tree).
- If x is not in P, then $P' = x, v_1, v_2, \ldots, v_k$ is a path in T that is strictly longer than P, contradicting our hypothesis that P is a longest path.

Hence every endpoint of a longest path in T is a leaf, and so T contains at least two leaves.