$\mathcal{NP}\text{-hard}$

A problem π is said to be \mathcal{NP} -hard if a polynomial time algorithm to solve π could be used to solve *every* problem in \mathcal{NP} in polynomial time.

Notes:

- If π satisfies $\forall \pi' \in \mathcal{NP}, \pi' \propto \pi$, then π is \mathcal{NP} -hard.
- ► If any NP-hard problem can be solved in polynomial time, then so can all problems in NP!
- Karp already proved that 3-SAT is NP-hard (as well as 20 other problems). Many other problems have been proven to be NP-hard.
- ► To prove that a new decision problem π is NP-hard, you just have to find a known NP-hard problem π' and show that π' ∝ π.