

2-colorability \propto 3-colorability

Let F map instances of 2-colorability to instances of 3-colorability as follows.

Given graph $G = (V, E)$, let $F(G)$ be the graph $G' = (V', E')$ defined by

- ▶ $V' = V \cup \{v^*\}$, where v^* is a new vertex
- ▶ $E' = E \cup \{(v^*, v) : v \in V\}$

It is not hard to see that F is a Karp reduction.

(In particular, G can be 2-colored if and only if $F(G)$ can be 3-colored.)

Yet, 2-colorability is in \mathcal{P} .

Do we learn anything about 3-colorability?