3-colorability: proving NO

Now suppose I have another large graph $G = (V, E)$, and my friend says it doesn't have a proper 3-coloring of its vertices.

How can my friend convince me that she is right?

Certainly she can list all the possible colorings and we can check them all (exhaustive search) to see if any of them are proper.

This would work, but it is really expensive.

Question to class: how expensive?